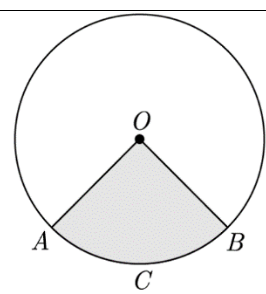
	INDIAN SCHOOL MUSCAT **HALF YEARLY EXAMINATION 2022** **MATHEMATICS** **MARKING SCHEME** **CLASS: 10** **SET A ,B ,C**	MKS:80
	1) a 2) a 3) c 4) a 5) b 6) d 7) a 8) c 9) d 10) d	1mk each
	11) F 12) F 13) T 14) F	1mk each
	15) +5 16) 45 17) 15/4 18) 154	1mk each
	19) d 20) b	1mk each
21	Assume that $2+5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$. $2+5\sqrt{3} = \frac{p}{q},\ q \neq 0$ $5\sqrt{3} = \frac{p}{q} - 2$ $5\sqrt{3} = \frac{p-2q}{q}$ $\sqrt{3} = \frac{p-2q}{5q}$ Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2+5\sqrt{3}$ is an irrational number. OR The required answer is the LCM of 9, 12, and 15 minutes. Finding prime factor of given number we have, $9 = 3 \times 3 = 3^2$ $12 = 2 \times 2 \times 3 = 2^2 \times 3$ $15 = 3 \times 5$ $\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$ $= 150$ minutes The bells will toll next together after 180 minutes.	½ 1 ½ 1 ½ ½
22	We have $2x - y = 2$ ...(1) $x + 3y = 15$ ...(2) From equation (1), we get $y = 2x - 2$ ...(3) Substituting the value of y in equation (2), $x + 6x - 6 = 15$ or, $7x = 21 \Rightarrow x = 3$ Substituting this value of x in (3), we get From equation (1), we have $y = 2 \times 3 - 2 = 4$ $x = 3$ and $y = 4$	½ 1 ½
23	Sum of zeroes, $\alpha + \beta = 3$ Product of zeroes $\alpha\beta = -\frac{2}{5}$ Now $p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$ $= x^2 - 3x - \frac{2}{5}$ $= \frac{1}{5}(5x^2 - 15x - 2)$ The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$	½ 1 ½

24	We have $x^2 + kx + 12 = 0$	
	If 2 is the root of above equation, it must satisfy it.	
	$(2)^2 + 2k + 12 = 0$	
	, $2k + 16 = 0$	
	$k = -8$	1
	Substituting $k = -8$ in $x^2 + kx + q = 0$ we have	
	$x^2 - 8x + q = 0$	½
	For equal roots,	
	$(-8)^2 - 4(1)q = 0$	
	$64 - 4q = 0$	½
	$4q = 64 \Rightarrow q = 16$	
	OR	
	We have $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$	½
	$\sqrt{2}\,x^2 + 2x + 5x + 5\sqrt{2} = 0$	½
	$\sqrt{2}\,x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$	
	$(x+\sqrt{2})(\sqrt{2}\,x+5) = 0$	
	Thus $x = -\sqrt{2}$ and $= -\frac{5}{\sqrt{2}}$	½ + ½
25	$\tan(3x + 30^\circ) = 1 = \tan 45^\circ$	1
	$3x + 30^\circ = 45^\circ$	½
	$x = 5^\circ$	½
26		
	$OA + OB + \overset{\frown}{ACB} = 31\text{ cm}$	
	$6.5 + 6.5 + \overset{\frown}{ACB} = 31\text{ cm}$	
	$\overset{\frown}{ACB} = 18\text{ cm}$	½
	Now, area of sector $OACBO$	
	$= \frac{1}{2} \times \text{radius} \times \overset{\frown}{ACB}$	½
	$= \frac{1}{2} \times 6.5 \times 18 = 58.5\text{ cm}^2$	½ + ½
	OR	
	Radius of circle $r = 10$ cm, central angle $= 90^\circ$	
	Area of minor segment,	
	$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^\circ\right]$	½ + ½
	$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5\text{ cm}^2$	½ + ½
27	$\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^\circ < \theta < 90^\circ$	
	$\sqrt{3}\sin\theta = \cos\theta$	½
	$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$	½
	$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$	½
	$\theta = 30^\circ$	½

28	We have, $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ By converse of BPT, $DE \parallel BC$ Due to corresponding angles we have $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ Given $\angle ADE = \angle AED$ Thus $\angle ABC = \angle ACB$ Therefore BAC is an isosceles triangle.	½ ½ ½ ½
29	We have $404 = 2 \times 2 \times 101$ $= 2^2 \times 101$ $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ $= 2^5 \times 3$ $\text{HCF}(404, 96) = 2^2 = 4$ $\text{LCM}(404, 96) = 101 \times 2^5 \times 3 = 9696$ $\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$ Also, $404 \times 96 = 38784$ Hence, $\text{HCF} \times \text{LCM} =$ Product of 404 and 96	½ ½ ½ ½ ½ ½
30	$\dfrac{4}{x} - 3 = \dfrac{5}{2x+3}$ $\dfrac{4}{x} - \dfrac{5}{2x+3} = 3$ $\dfrac{4(2x+3) - 5x}{x(2x+3)} = 3$ $8x + 12 - 5x = 3x(2x+3)$ $3x + 12 = 6x^2 + 9x$ $6x^2 + 6x - 12 = 0$ $x^2 + x - 2 = 0$ $x^2 + 2x - x - 2 = 0$ $x(x+2) - (x+2) = 0$ $(x+2)(x-1) = 0$ Thus $x = 1, -2$ OR We have $x^2 + 6x - (a^2 + 2a - 8) = 0$ Comparing with $Ax^2 + Bx + C = 0$ we get $A = 1, B = 6, C = (a^2 + 2a - 8)$ The roots are given by the quadratic formula $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ $= \dfrac{-6 \pm \sqrt{36 + 4(a^2 + 2a - 8)}}{2}$ $= \dfrac{-6 \pm (2a+2)}{2}$ Thus $x = \dfrac{-6 + (2a+2)}{2} = a - 2$ and $x = \dfrac{-6 - (2a+2)}{2} = -a - 4$ Thus $x = a - 2, -a - 4$	½ ½ ½ 1 ½ + ½ ½ ½ ½ ½ ½

31

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$(\tan A + \cot A)^2 = (2)^2 \qquad \text{½}$$
$$\tan^2 A + \cot^2 A + 2\tan A \cot A = 4 \qquad \text{½}$$
$$\tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} = 4 \qquad \text{½}$$
$$\tan^2 A + \cot^2 A + 2 = 4 \qquad \text{½}$$
$$\tan^2 A + \cot^2 A = 4 - 2 \qquad \text{½}$$
$$\tan^2 A + \cot^2 A = 2 \qquad \text{½}$$

OR

$$\text{LHS} = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$
$$= \frac{\cos A}{1 - \left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1 - \left(\frac{\cos A}{\sin A}\right)} \qquad \text{½}$$
$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \qquad \text{½}$$
$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \qquad \text{½}$$
$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \qquad \text{½}$$
$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)} \qquad \text{½}$$
$$= \cos A + \sin A \qquad \text{½}$$
$$= \sin A + \cos A$$
$= \text{RHS}$ Hence proved.

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In ΔABC and ΔACD we have

$\angle ACB = \angle CDA$ [given]

$\angle CAB = \angle CAD$ [common]

By AA similarity criterion we get

$\Delta ABC \sim \Delta ACD$ 1

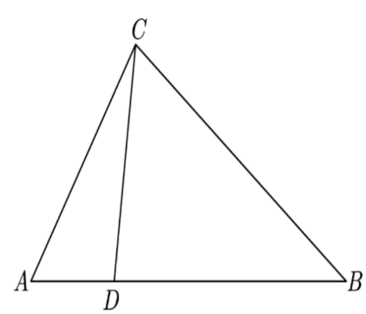
Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$ ½

Now $\frac{AB}{AC} = \frac{AC}{AD}$ ½

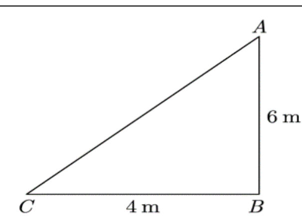
$AC^2 = AB \times AD$ ½

$6^2 = AB \times 3$

$AB = \frac{36}{3} = 12 \text{ cm}$ ½



Again, let PQ be height of pole and QR be its shad the same time, the angle of elevation of tree and pc equal i.e $\Delta ABC \sim PQR$ 1 DIAG ½

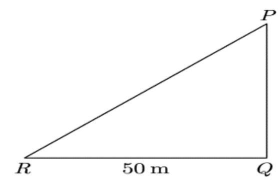


$$\frac{AB}{BC} = \frac{PQ}{QR}$$ ½

$$\frac{6}{4} = \frac{PQ}{50}$$ ½

$$PQ = \frac{50 \times 6}{4}$$

$$= 75 \text{ m}$$ ½



33

(i) We have used AA similarity criterion. 1

(ii) Here, $\angle ABC = \angle DEC$ (90° each)
Since vertical opposite angle are equal,

$$\angle ACB = \angle DCE$$ 1

Thus due to AA similarity criterion,

$$\Delta ABC \sim \Delta DEC$$

and $\angle BAC = \angle CDE$
Therefore both are correct.
Thus (a) is correct option.

(iii)
We have $\frac{AB}{DE} = \frac{BC}{CE}$ 1

$$\frac{60}{x} = \frac{50}{40} \Rightarrow x = 48 \text{ ft}$$ 1

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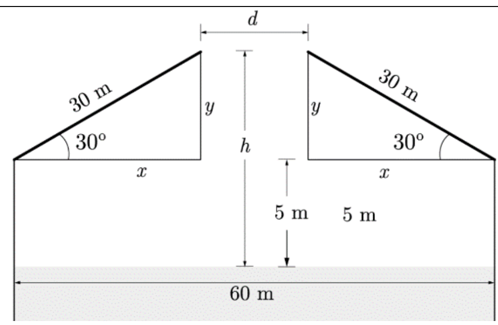


Now $\sin 30^\circ = \frac{y}{30}$

$$\frac{1}{2} = \frac{y}{30}$$

$$y = \frac{30}{2} = 15 \text{ m}$$ 1

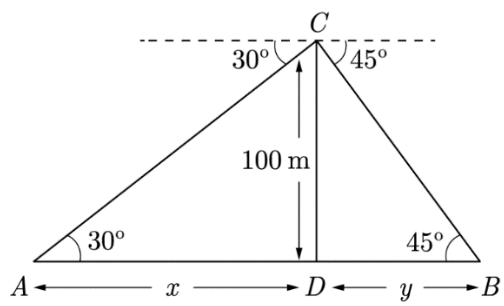
$$h = y + 5 = 15 + 5 = 20 \text{ m}$$

Now $\cos 30^\circ = \frac{x}{30}$

$$\frac{\sqrt{3}}{2} = \frac{x}{30}$$

$$x = \frac{30\sqrt{3}}{2} = 15\sqrt{3} \text{ m}$$ 1

	$\cos 30^\circ = \frac{x}{30}$ $\frac{\sqrt{3}}{2} = \frac{x}{30}$ $x = \frac{30\sqrt{3}}{2} = 15\sqrt{3}$ m $d = 60 - x - x$ $= 60 - 15\sqrt{3} - 15\sqrt{3}$ $= 60 - 30\sqrt{3} = 30(2 - \sqrt{3})$ $= 30(2 - 1.732)$ $= 30 \times 0.268 = 8.04$ m	½ ½ ½ ½
35	Let c represent the speed of the current and s represent the cruising speed of the ship. Trip to the Goa, $(s+c)70 = 1435$ $2s+2c = 41 \quad (1)$ Return to Mumbai, $(s-c)82 = 1435$ $2s-2c = 35 \quad (2)$ Adding eq (1) and (2) we have $4s = 76 \Rightarrow s = 19$ kmph Subtracting eq (2) from (1) we have $4c = 6 \Rightarrow s = 1.5$ kmph	1 1 1 1
36	Let the sides of two squares be a and b, then $a^2 + b^2 = 400 \quad (1)$ and $4(a-b) = 16$ $a - b = 4 \Rightarrow a = 4 + b$ From equations (1) and (2), we obtain $(4+b)^2 + b^2 = 400$ $16 + b^2 + 8b = b^2 = 400$ $2b^2 + 8b - 384 = 0$ $b^2 + 4b - 192 = 0$ $b^2 + 16b - 12b - 192 = 0$ $b(b+16) - 12(b+16) = 0$ $(b+16)(b-12) = 0$ $b = -16, 12$ Rejecting the negative value, $b = 12$ cm then $a = 4 + 12 = 16$ cm.	½ ½ ½ ½ 1 ½ ½ ½

<table>
<tr><td></td><td>OR
Let numerator be x, then denominator will be $x+2$.
and fraction $= \frac{x}{x+2}$
Now $\frac{x}{x+2} + \frac{x+2}{x} = \frac{34}{15}$
$15(x^2 + x^2 + 4x + 4) = 34(x^2 + 2x)$
$30x^2 + 60x + 60 = 34x^2 + 68x$
$4x^2 + 8x - 60 = 0$
$x^2 + 5x - 3x - 15 = 0$
$x(x+5) - 3(x+5) = 0$
$(x+5)(x-3) = 0$ $\therefore x = -5 \text{ or } x = 3$
$fraction = \frac{3}{5}$</td><td>½
½
½
1
½
½ + ½
1</td></tr>
<tr><td>37</td><td>
Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.
In right ΔADC,
$\tan 30^\circ = \frac{CD}{AD}$
$\frac{1}{\sqrt{3}} = \frac{100}{x}$
$x = 100\sqrt{3}$ ...(1)
In right ΔBDC,
$\tan 45^\circ = \frac{CD}{DB}$
$1 = \frac{100}{y} \Rightarrow y = 100 \text{ m}$</td><td>1 DIAG
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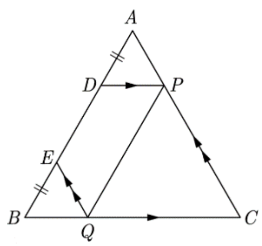
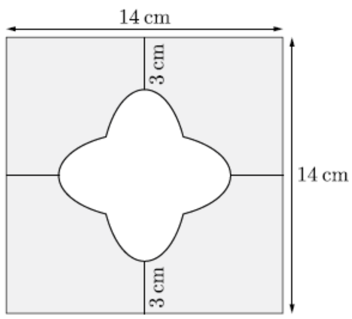
	Distance between two cars $AB = AD + DB = x + y$ $= (100\sqrt{3} + 100)$ $= (100 \times 1.73 + 100)$ m $= (173 + 100)$ m $= 273$ m Hence, distance between two cars is 273 m.	1 ½ ½
38	 In ΔABC, $DP \parallel BC$ By BPT we have $\frac{AD}{DB} = \frac{AP}{PC}$, ...(1) Similarly, in ΔABC, $EQ \parallel AC$ $\frac{BQ}{QC} = \frac{BE}{EA}$ ...(2) From figure, $EA = AD + DE$ $= BE + ED$ $(\ BE = AD)$ $= BD$ Therefore equation (2) becomes, $\frac{BQ}{QC} = \frac{AD}{BD}$ ...(3) From (1) and (3), we have $\frac{AP}{PC} = \frac{BQ}{QC}$ By converse of BPT, $PQ \parallel AB$ Hence Proved	1 ½ 1 ½ ½ ½ + ½
39.	$l(minor\ arc) = \frac{\theta}{360} \times 2\pi r == \frac{90}{360} \times 2 \times 3.14 \times 10 = 15.7cm$	1
	Area of sector $OAPB$, $= \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$ Area of ΔAOB, $= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$ Area of minor segment $AQBP$, $= (25\pi - 50) \text{ cm}^2$ $= 25 \times 3.14 - 50$ $= 78.5 - 50 = 28.5 \text{ cm}^2$ Also area of circle $= \pi(10)^2$ $= 3.14 \times 100 = 314 \text{ cm}^2$ Area of major segment $ALBQA = 314 - 28.5$ $= 285.5 \text{ cm}^2$ OR	1 1 1 1 ½ ½

	$3+r+2r+r+3 = 14$	
	$4r+6 = 14 \Rightarrow r=2$	1
	Thus radius of the semi-circle formed inside is 2 cm and length of the side of square formed inside the semi-circle is 4 cm.	
	Area of square $ABCD$	
	$= 14 \times 14 = 196 \text{ cm}^2$	1
	Thus area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$	
	$= 2 \times 3.14 \times 2 \times 2 = 25.12 \text{ cm}^2$	1
		
	Area of the square formed inside the semi-circle	
	$(2r)^2 = 4 \times 4 = 16 \text{ cm}^2$	1
	Area of the shaded region,	
	$=$ area of square $ABCD$	
	$-$ (Area of 4 semi-circle +Area of square)	
	$= 196 - (25.12 + 16)$	½
	$= 196 - 41.12 = 154.88 \text{ cm}^2$	½